

# Collective Coordinates for D-branes

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## Abstract

We develop a formalism for the scattering off D-branes that includes collective coordinates. This allows a systematic expansion in the string coupling constant for such processes, including a worldsheet calculation for the D-brane's mass.

## 1 Introduction

In the recent developments of string theory, solitons play a central role. For example, in the case of duality among various string theories, solitons of a weakly coupled string theory become elementary excitations of the dual theory [1]. Another example, among many others, is the role of solitons in resolving singularities in compactified geometries, as they become light [2].

In order to gain more insight in the various aspects of these recent developments, it is important to understand the dynamics of solitons in the context of string theory. In this quest one of the tools at our disposal is the use of scattering involving these solitons. This includes the scattering of elementary string states off these solitons as well as the scattering among solitons. An important class of solitons that have emerged are D-branes [4]. In an influential paper [3] it was shown that these D-branes carry  $R - R$  charges, and are therefore a central ingredient in the aforementioned dualities. These objects are described by simple conformal field theories, which makes them particularly suited for explicit calculations.

In this paper we use some of the preliminary ideas about collective coordinates developed in a previous paper [5], to describe the scattering of elementary string states off D-branes. We clarify and expand these ideas, providing a complete analysis of this process.

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The paper begins by reviewing the presence of non-local violations of conformal invariance that are a consequence of the existence of translational zero modes. It is then shown how the world sheet theory removes these anomalies through the introduction of collective coordinates (which appear as “wormhole parameters”), and the inclusion of the contribution of disconnected worldsheets to the S-matrix elements. As a byproduct of this analysis we calculate the mass of the soliton by a purely worldsheet approach (that is amenable to higher order corrections). We thereby recover a result obtained in [4] using effective low energy considerations.

These ideas are then applied to the example of a closed string state scattering off a 0-brane. We show how to include the recoil of the brane, and discover the interpretation of the wormhole parameters as collective coordinates.

The conclusion contains some comments on higher order corrections to scattering and to mass renormalization as well as possible extensions of this work in different directions.

## 2 Conformal anomalies.

D-branes placed at different locations in space have the same energy. This implies the existence of spacetime zero modes, one for each spatial direction transverse to the D-brane (i.e. one for each Dirichlet direction). Because of these zero modes, the attempts to calculate scattering matrix elements in a weak coupling expansion encounter spacetime infra-red divergences. In point-like field theory these divergences arise from zero modes propagating in Feynman diagrams (for example diagrams as in Fig. 1 , where  $\eta_0$  is a zero mode).

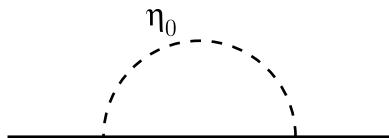


Fig. 1: Diagram containing the propagation of a zero-mode.

Such behavior can also be found in the canonical formalism, where it manifests itself by the presence of vanishing energy denominators, characteristic of degenerate perturbation theory. This is resolved in quantum field theory by the introduction of collective coordinates. The right basis of states, in which the degeneracies are lifted, are the eigenstates of momenta conjugate to these coordinates.

In string theory, however, we do not have a workable second quantized formalism. Instead, we have to rely on a first quantized description. The weak coupling expansion consists of histories spanning worldsheets of successively higher topologies. Yet, the aforementioned divergences are still present, and are found in a specific corner of moduli space. The divergences appear as violations of conformal invariance, as described below. To the lowest order, these singularities are found when one considers worldsheets as depicted in Fig. 2.

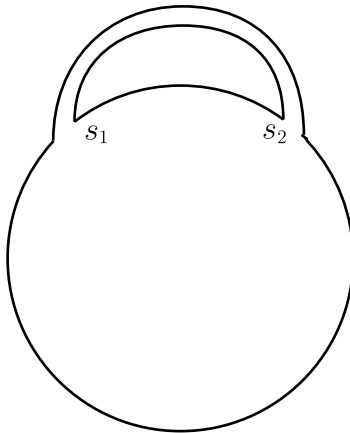


Fig. 2: The surface  $\Sigma$ .

This surface,  $\Sigma$ , is obtained by attaching a very long and thin strip to the boundary of the disc. This is equivalent to removing two small segments of the boundary of the disc and identifying them, thus creating a surface with the topology of an annulus. In the degeneration limit, when the strip becomes infinitely long and thin, the only contribution comes from massless open string states propagating along the strip. In the case of Dirichlet boundary conditions, the only such states are the translational zero modes.

In the case of closed string solitons, the divergence appears for degenerate surfaces containing long thin handles, along which the zero mode, a closed string state in that

case, propagates. In both cases these limiting world-histories are reminiscent of the field theory Feynman diagrams, as in Fig. 1 .

We are now ready to properly extract the divergences. As discussed in [7] for the closed bosonic string theory, the matrix element in the degeneration limit can be obtained by a conformal field theory calculation on the lower genus surface, with operators inserted on the degenerating segments (see Fig. 3 ).

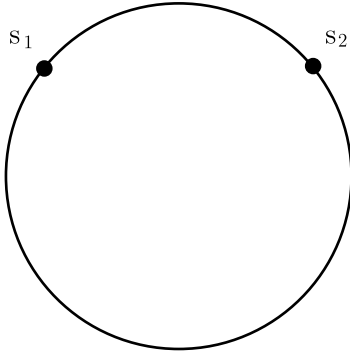


Fig. 3: The degenerations limit of the surface  $\Sigma$ .

This can be expressed formally as follows:

$$\langle V_1 \dots V_n \rangle_\Sigma = \sum_a \int \frac{dq}{q} q^{-2h_a} \langle \phi_a(s_1) \phi_a(s_2) V_1 \dots V_n \rangle_{\text{Disc}} \quad (2.1)$$

where  $\{\phi_a\}$  is a complete set of eigenstates of  $L_0$  (which generates translations along the strip), with weights  $h_a$ . The limit  $q \rightarrow 0$ , therefore, projects the sum into the lowest weight states.

As an illustration, it is useful to first consider the same limit for the usual open bosonic string (with Neumann boundary conditions). The lowest weight states (excluding the tachyon) are:

$$\phi_k^i = N_k \sqrt{g} \left( \frac{dX^i}{ds} \right) e^{ikX} \quad (2.2)$$

with  $h_k = \frac{k^2}{2}$  and  $N_k$  a normalization factor. Using the above formula (2.1), one gets:

$$\langle V_1 \dots V_n \rangle_\Sigma = \sum_i \int \frac{d^d k}{(2\pi)^d} \int dq q^{-(1+k^2)} \langle \phi_k^i(s_1) \phi_k^i(s_2) V_1 \dots V_n \rangle_{\text{Disc}} . \quad (2.3)$$

This expression exhibits the exchange of a photon, where  $\log q$  is the time parametrizing the photon worldline. The requirement of factorization fixes the normalization of the operators  $\phi_a$ .

We now turn to the analysis for the D-branes, which is rather similar. The Dirichlet p-brane is defined by imposing Neumann boundary conditions on  $X^N = X^0 \dots X^p$ , and Dirichlet boundary conditions on the remaining coordinates,  $X^D$ . The lightest states (excluding the tachyon) are the translational zero modes of the D-brane:

$$\phi_i = N \partial_n X_i^D e^{i\omega X_0} \quad i = p+1, \dots, d-1. \quad (2.4)$$

where  $X_0$  is target time,  $\partial_n$  is the derivative normal to the boundary, and  $N$  is a normalization constant. The existence of these zero modes reflects the degeneracy of different D-branes configurations, obtained by rigid translations ( $\vec{k}_N = 0$ ) in the Dirichlet directions.

In order to allow a general p-brane ( $p > 0$ ) to recoil one needs to compactify the spatial Neumann directions and wrap the p-brane around them. In that case the mass of the p-brane is finite and proportional to the spatial Neumann volume. For the sake of clarity we focus on the scattering off a 0-brane, where no such compactification is necessary.

The normalization constant  $N$  that appears in (2.4) is determined by factorization; consider the S-matrix element involving two closed string tachyons whose vertex operators are:

$$\begin{aligned} V_1 &= N_t \int d^2 z_1 e^{i\omega_1 X_0(z_1) - i\vec{k}_1^D \cdot \vec{X}^D(z_1)} \\ V_2 &= N_t \int d^2 z_2 e^{i\omega_2 X_0(z_2) - i\vec{k}_2^D \cdot \vec{X}^D(z_2)}. \end{aligned} \quad (2.5)$$

The factorization on open string poles is obtained by letting one of the tachyons approach the boundary (see Fig. 4).

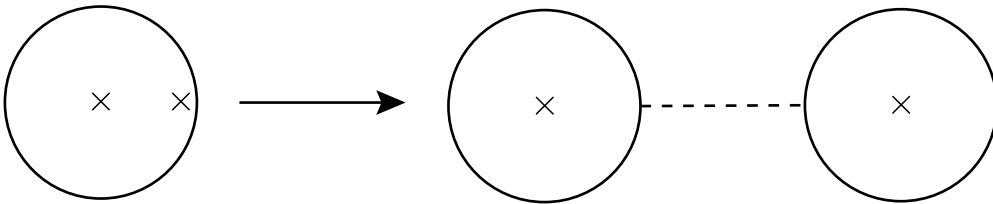


Fig. 4: Factorization on the Disc.

In the factorization one recognizes the various open string poles, in particular the tachyon at  $\omega^2 = -\frac{1}{2}$ , and the zero mode at  $\omega^2 = 0$ . This yields the normalization factor

for the zero mode:<sup>1</sup>

$$N = \frac{\sqrt{g}}{4}. \quad (2.6)$$

Since these states don't carry Dirichlet momenta, they are discrete quantum mechanical states.

We are now ready to extract the violations of conformal invariance for S-matrix elements calculated on the annulus. As mentioned above, these divergences appear in the limit of moduli space described in Fig. 2. In this limit the divergent contribution to an S-matrix element involving  $n$  elementary string states is:

$$\left\langle \int V_1 \dots \int V_n \right\rangle_{\Sigma} = \frac{-g}{16T} \log \epsilon \left\langle \partial_n \vec{X}_D(s_1) \cdot \partial_n \vec{X}_D(s_2) \int V_1 \dots \int V_n \right\rangle_{\text{Disc}} \quad (2.7)$$

where  $T$  is a large target time cut-off, and  $\epsilon$  is a world-sheet cut-off. The appearance of  $T$  is necessary in order to properly extract the divergent contributions due to the translational zero modes.

### 3 Wormhole Parameters

In order to restore conformal invariance we appeal to the Fischler-Susskind mechanism [6]. The idea is to modify the two-dimensional action such that the sum of the various world-sheet contributions to the S-matrix element is conformally invariant.

The Fischler-Susskind mechanism requires in this case the addition of a non-local operator  $O$  to the world-sheet action:

$$O = \frac{g}{8T} \log^2 \epsilon \int ds_1 \partial_n \vec{X}_D(s_1) \int ds_2 \partial_n \vec{X}_D(s_2). \quad (3.1)$$

This operator is a bilocal involving the translational zero mode vertex operators. Notice the additional logarithmic divergence beyond the one that is already present in (2.7). This additional divergence accounts for the volume of the residual Möbius subgroup that fixes the locations of two vertex operators:

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<sup>1</sup>In our notation the open string tachyon is

$$\frac{1}{2} \sqrt{g} e^{i\omega X_0}$$

$$\frac{1}{V_{\text{Möbius}}} \left\langle \int ds_1 \partial_n X(s_1) \int ds_2 X(s_2) \int V \dots \int V \right\rangle_{\text{Disc}} = \frac{1}{V_{\text{Residual}}} \left\langle \partial_n X(s_1) \partial_n X(s_2) \int V \dots \int V \right\rangle_{\text{Disc}} \quad (3.2)$$

where  $V_{\text{Residual}} = -2 \log \epsilon$  is the residual Möbius volume as defined above [11]. In the case of the Fischler-Susskind mechanism involving a local operator this would correspond instead to a finite factor.

In order to rewrite the sum of histories as an integral weighted by a local action we introduce wormhole parameters  $\alpha_i$ , one for each Dirichlet direction. As will become clear these parameters are proportional to the momenta conjugate to the center of mass position of the 0-brane.

The S-matrix element then takes the form:

$$S = \int d\vec{\alpha} e^{-\vec{\alpha}^2/2} \int DX e^I \int V_1 \dots \int V_n \quad (3.3)$$

$$I = \frac{-1}{4\pi} \int d^2 z (\partial \vec{X} \cdot \bar{\partial} \vec{X} + \partial X^0 \cdot \partial X^0) + \frac{\vec{\alpha} \sqrt{g}}{2\sqrt{T}} \log \epsilon \int ds \partial_n \vec{X} + \frac{1}{8\pi} \int ds \vec{a}(X_0) \partial_n \vec{X} \quad (3.4)$$

As will be shown below, the inclusion of the last term in the action is important in restoring conformal invariance. At the lowest order in the string coupling constant  $g$ :

$$\vec{a}(X_0) = \vec{a}_0$$

where  $\vec{a}_0$  is a constant (independent of  $X_0$ ), and represents the fixed location of the brane.

## 4 Scattering off a 0-brane

In organizing various contributions that violate conformal invariance we begin by considering the “vacuum persistence amplitude”:

$${}_{T/2} \langle 0 | 0 \rangle_{-T/2}^{\vec{\alpha}, \vec{a}} = \int DX \exp \left\{ \frac{-1}{4\pi} \int \partial X \bar{\partial} X + \frac{\vec{\alpha} \sqrt{g}}{2\sqrt{T}} \log \epsilon \int \partial_n \vec{X} + \frac{1}{8\pi} \int \vec{a}(X_0) \partial_n \vec{X} \right\}. \quad (4.1)$$

This expression differs from the vacuum persistence amplitude in the absence of  $\vec{\alpha}$ ,  $\vec{a}$ , as can be seen in the following expression:

$$\begin{aligned} {}_{T/2}\langle 0|0\rangle_{-T/2}^{\vec{\alpha},\vec{a}} &= \exp\left(-\frac{\alpha^2}{T} \log \epsilon\right) \exp\left(\frac{\dot{a}^2}{\pi^2 g} \log \epsilon\right) \\ &\quad \exp\left(-\frac{1}{2\pi\sqrt{gT}} \vec{a}_0 \cdot \vec{\alpha}\right) {}_{T/2}\langle 0|0\rangle_{-T/2}^{\vec{\alpha}=0,\vec{a}=\vec{a}_0} \end{aligned} \quad (4.2)$$

In order to explain equation (4.2), we note first that this contribution to the S-matrix element shows up diagrammatically as the sum of disconnected diagrams (see Fig. 5). These disconnected diagrams have to be added to the more conventional connected diagrams in order to restore conformal invariance<sup>2</sup>

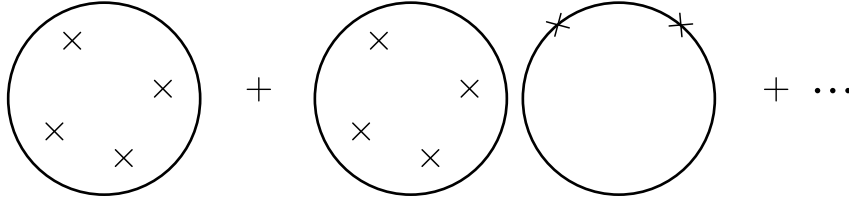


Fig. 5: The sum over disconnected diagrams.

The various terms appearing in the exponent in eq. (4.2) are calculated on the disk as the two-point function of the operator  $O'$ :

$$O' = \int ds \left( \frac{\vec{\alpha}\sqrt{g}}{2\sqrt{T}} \log \epsilon \partial_n \vec{X} + \frac{1}{8\pi} \vec{a}(X_0) \partial_n \vec{X} \right).$$

Care has to be exercised to carefully account for the Möbius volume associated to the disc. The S-matrix element then becomes:

$$\begin{aligned} S &= \int d\vec{\alpha} e^{-\vec{\alpha}^2/2} \int DX \exp\left(-\frac{\vec{\alpha}^2}{T} \log \epsilon\right) \exp\left(\frac{\dot{a}^2}{\pi^2 g} \log \epsilon\right) \\ &\quad \exp\left(-\frac{1}{2\pi\sqrt{gT}} \vec{a}_0 \cdot \vec{\alpha}\right) e^I \int V_1 \dots \int V_n. \end{aligned} \quad (4.3)$$

For simplicity we analyze the scattering of a closed string tachyon off the D-brane. The initial and final momenta are respectively  $\vec{k}_1$  and  $\vec{k}_2$  with excess momentum  $\vec{P} = \vec{k}_1 + \vec{k}_2$ .

The S-matrix element (4.3) is then, after some straightforward algebra:

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<sup>2</sup>Disconnected diagrams have previously been considered in the case of the D-instanton [9].



$$\begin{aligned}
S = & \int d\vec{\alpha} e^{-\vec{\alpha}^2/2} \exp \left\{ \frac{\vec{a}_0}{2} \cdot \left( i\vec{P} - \frac{\vec{\alpha}}{\pi\sqrt{gT}} \right) \right\} \\
& \exp \left\{ - \left( \frac{\vec{\alpha}}{\sqrt{T}} - i\pi\sqrt{g}\vec{P} \right)^2 \log \epsilon \right\} \\
& \exp \left\{ \left( \frac{\vec{a}^2}{\pi^2 g} - \pi^2 g \vec{P}^2 \right) \log \epsilon \right\} \\
& \exp \left\{ \frac{1}{2} \left( \omega_1 + \omega_2 + \frac{1}{2} \vec{a} \cdot \vec{P} \right)^2 \log \epsilon \right\} F(\vec{\alpha}, \vec{k}_1, \vec{k}_2, \vec{a}) \quad (4.4)
\end{aligned}$$

where  $F$  is the finite part of the S-matrix element, up to order  $g^0$ , evaluated using the action (3.4).

Conformal invariance is restored only if:

$$\vec{\alpha} = i\pi\vec{P}\sqrt{gT} \quad (4.5)$$

$$\vec{a} = \pi^2 g \vec{P} \quad (4.6)$$

$$\omega_1 + \omega_2 + \frac{1}{2} \vec{a} \cdot \vec{P} = 0 \quad (4.7)$$

equation (4.5) identifies  $\vec{\alpha}$  as being proportional to the recoil momentum; equation (4.6) identifies  $\vec{a}$  as the recoil velocity (interpreting the coefficient as the inverse mass); equation (4.7) expresses the conservation of energy; using (4.6) it can be written as:

$$\omega_1 + \omega_2 + \frac{\vec{P}^2}{2M} = 0. \quad (4.8)$$

This can also be obtained from (4.4) by integrating over the constant part of  $X_0$ . The S-matrix element then becomes:

$$S = \exp \left( -\frac{1}{2} \pi^2 g \vec{P}^2 T \right) \delta(\omega_1 + \omega_2 + \frac{1}{2} \pi^2 g \vec{P}^2) G(\vec{k}_1, \vec{k}_2) \quad (4.9)$$

where  $G(\vec{k}_1, \vec{k}_2)$  is obtained from  $F$  above by substituting the above values for  $\vec{\alpha}, \vec{a}$ , and contains in particular the contribution from intermediate, virtually recoiling, brane.

Note that relation (4.6), that enforces momentum conservation, eliminates the dependence on  $\vec{a}_0$ .<sup>3</sup> We therefore conclude that (4.10) represents the transition from initial D-brane at rest, to a final state in which the D-brane has recoil momentum  $\vec{P}$ . This allows us to identify:

$$M = \frac{1}{\pi^2 g}$$

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<sup>3</sup>Similarly, the conformal violations proportional to  $\vec{a}$  cancel, at this order, when adding connected and disconnected diagrams.

which is consistent with the results of [4].

## 5 Conclusion

In conclusion, we have applied the Fischler-Susskind mechanism to the case of D-brane recoil. This approach can be systematically extended to higher orders in string perturbation theory. Contributions from disconnected diagrams have to be included in order to restore conformal invariance and translational invariance. A purely two-dimensional calculation of the soliton mass is obtained.

In this paper we consider scattering of elementary string state off the D-brane. This still leaves the question is the scattering among D-branes [12]. Since D-branes are BPS saturated one might hope to find the metric on the moduli space describing such scattering ( in analogy to the work of [8] on BPS monopoles). This would entail finding the space on which the wormholes move.

Another interesting question is the treatment of fermionic collective coordinates, from a two-dimensional viewpoint, and the appearance of a manifestly supersymmetric collective coordinate description.

In summary, a better understanding of the dynamics of D-branes can shed more light on the relations among string theories.

## Acknowledgments

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After the completion of this work we have received a paper dealing with similar issues [10]. We disagree with the conclusions of that paper.

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